Challenges Pupils Face in Enlargement Transformation Geometry in Zambian Secondary Schools: A Case of Two Schools in Lusaka and Monze Districts

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Abstract

Zambian Secondary Schools have experienced a low performance in Mathematics at School Certificate level in recent years (Examination Council of Zambia (ECZ), 2016). An analysis of results in terms of topics in the examinations showed that Transformation geometry is one of the topics in mathematics in which pupils have not been performing very well (ECZ, 2009; ECZ, 2008). For this reason, this study was set to establish the challenges pupils faced in learning transformation geometry in particular enlargement transformation geometry in two schools in Zambia. School A is located in Lusaka District and School B is in Monze district. The Van Hiele Model of levels of thought were used as the theoretical framework of the study. The study largely used a mixed method approach that included descriptive study design. The data was obtained through written test and interview schedules. One hundred Grade 12 pupils, all together from both schools wrote the test and six volunteered to undergo interview. Data was analysed through a process of coding, categorizing, clustering and performance indicators corresponding to the Van Hiele's model were used in the analysis of data. The study findings revealed that pupils faced challenges in solving enlargement transformation geometry problems. The study demonstrated that most of the grade 12 pupils were reasoning at the lowest level (visualisation) of the Van Hiele's model of geometric thought.

Pupils had problems with the concept of the centre and scale factor of an enlargement. The findings revealed that pupils demonstrated ignorance of the basic relationships of figures under enlargement through identifying incorrect image of the triangular figure given. Pupils expressed challenges in explaining the steps they took to obtain the image after enlargement because they did not have enough pre-requisite knowledge on the topic.

The study concluded that pupils lacked relevant and enough pre-requisite knowledge to enlargement transformation, and they lacked exposure to more geometrical problem-solving situations. The study concluded that teachers should strive to motivate learners on the topic and equip them with necessary pre-requisite knowledge on enlargement transformation. This should include introducing pupils to more practical situational problem-solving tasks which should involve explanation as well.

Key words: Challenges, Enlargement Transformation, Problem-solving, Spacial skills.

Introduction

According to ECZ (2008), the examiners' report showed that the questions on statistics and geometric graphs were well done by most pupils, while questions on transformations, trigonometry and earth geometry were poorly done. Furthermore, ECZ (2009) indicated that the geometry topic in which pupils performed poorly, was Transformation Geometry, as compared to other geometry topics. However, the major goal of secondary school geometry is to develop in individuals, the power of mathematical reasoning and abilities to be aware of the real world and be able to solve real life problems. As such, to help pupils achieve this goal, the secondary school mathematics syllabus suggests that reasoning about shapes should involve the use of coordinates and transformation techniques. While keeping this urge in place, most of the reports compiled by ECZ indicated that pupils seemed to lack geometric skills and understanding. This means that pupils may lack enough acquisition of geometric skills such as the ability to imagine, rotate, slide, reflect, enlarge, shear, and stretch an object which are essential in facilitating the learning of Transformation Geometry and other geometrical concepts. However, the ECZ review report of 2012 indicated that pupils appeared to have problems with skills that involve the concept of enlargement.

The problems pupils experienced in Enlargement Transformation Geometry could be explained and understood by considering the study done by Van Hiele (1999). Van Hiele's research gained its roots in Piaget's work which earlier identified four stages of cognitive development namely; the sensorimotor, the preoperational, the concrete operational and the formal operational stage (Piaget, 2001). Van Hiele's Model focused primarily on five levels of geometric conceptualization, namely; visualisation, analysis, abstraction, deduction and rigor. Van Hiele (1999), in his work Teaching Children Mathematics, suggested that learners advanced through these levels of thought in geometry learning. However, he argued that if pupils bypassed any given level, they might not perform well in the subsequent higher levels. Furthermore, he contended that a pupil might have misconceptions and misunderstandings if the geometry material presented was at a higher level than that of the pupil.

The findings of Van Hiele about geometric conceptualisation were used to explain the reasons why many secondary school pupils were having problems in geometry learning. Literature further revealed that these levels of geometric understanding in van Hiele's model were an accurate means of evaluating pupils' readiness for formal geometry instruction.

In this regard, it seems that the reasons for pupils' challenges to fully understand concepts of enlargement have not been fully given adequate research attention in Zambia, hence this study was tailored to investigate into the challenges which pupils were experiencing in learning Enlargement Transformation Geometry in the two selected schools.

Statement of the Problem

Literature shows that many pupils have tremendous misconceptions concerning a number of important geometry ideas. In Zambia, the examiners' report showed that the questions on statistics and geometric graphs were well done by most pupils, while questions on transformations, trigonometry and earth geometry were poorly done (ECZ, 2008). According to ECZ (2009), pupils performed poorly in Transformation Geometry as compared to other geometry topics. In addition, ECZ (2012) held that pupils faced a variety of difficulties relating to Enlargement Transformation Geometry in mathematics, especially those that recorded fail grade in the subject (Ibid, 2012). It was further observed that while some pupils in Zambia were very successful in solving problems on transformation geometry, many of them solved the problems algorithmically with little or no understanding of what was behind the procedures and calculations (Op cite, 2012). Difficulties with proofs also appear in the learning of Enlargement Transformation Geometry in Zambian secondary schools (ECZ, 2016). Most of the reports compiled by ECZ indicated that most pupils seemed to lack geometric skills and understanding. What was not known were the challenges which pupils faced in learning enlargement transformation geometry in line with the Van Hiele's Model of geometry learning in the secondary schools in Zambia.

Purpose of the Study

The purpose of the study was to determine the challenges which pupils had in learning enlargement transformation geometry in line with the Van Hiele's Model of geometry learning.

Objectives of the study

The study was guided by the following objectives:

(i) Ascertain pupils' levels of difficulty in enlargement transformation geometry learning
(ii) Determine the root causes of the challenges that pupils face in solving enlargement transformation geometry problems.

Research Questions

The above objectives were addressed by answering the following questions:

(i) What were pupils' levels of difficulty in enlargement transformation geometry learning
(ii) What was causing the challenges that pupils faced in solving enlargement transformation geometry problems?

Theoretical Framework.

The Van Hiele's model was used to describe and explain the findings of the study. Corley (1990) argued that the levels of geometry understanding in Van Hiele's model are an accurate means of evaluating pupils' readiness for formal geometry instruction. Van Hiele's research which has its roots in Piaget's work focused primarily on five levels of geometric conceptualisation. Van Hiele (1999), in his work Teaching Children Mathematics, suggested that learners advanced through levels of thought in geometry learning. These levels are characterised by visualisation (Basic level), which is the first level. Pupils under this level are usually only aware of shapes as a whole, and not of their properties or of their components (Jones, 1998). Concerning transformation geometry, pupils should recognize a particular transformation with reference to the changes in the figure and motion, without explicit regard to their properties of its components. The second level is Analysis (Level 1), and pupils are expected to analyse figures in terms of their components and relationships between components and discover properties or rules empirically (Jones, 1998). In transformation geometry, pupils operating under analysis level are expected to discover properties of change due to actions of a transformation.

The level of Abstraction (Level 2), is the third level in which the pupils are expected to form definitions of shapes based on their common properties. Under this level, pupils can create meaningful definitions and give informal arguments to justify their reasoning. The role and significance of formal deduction, however, is not understood, but they can establish inter-relationships between networks of theorems (Jones, 2002). At abstraction level, regarding transformation geometry, pupils are expected to inter-relate the properties of change due to a particular action of transformation. The fourth level is Deduction (Level 3). Under this level, pupils understand the role of axioms/postulation systems and definitions and know the meaning of necessary and enough conditions. In transformation geometry, pupils are expected to perform transformation geometry proofs using transformation approach, think through and give reasons in multi-steps problem. Rigor is the fifth level (Level 4). Pupils at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. These pupils can understand the use of indirect proof and proof by contradiction. However, the study did not appeal to the level of Rigor because the Zambian secondary school Mathematics curriculum does not include aspects of transformation geometry under this level.

Hence the challenges experienced by pupils in enlargement transformation geometry could be explained and understood better by reflecting on the research work done by Van Hiele (1999) based on the sequential stages of geometry learning.

LITERATURE REVIEW

There is limited availability of research on pupils' understanding and learning of transformation geometry (Sarah and Javaluxmi, 2012). In this regard, Edwards (1997) explained that Transformation Geometry provided an opportunity for pupils to develop their spacial visualisation skills and geometrical reasoning abilities. According to Ilaslan (2013), Transformation Geometry is a subset of geometry in which pupils learn to identify and illustrate movement of shapes. Transformation Geometry is a dynamic approach to learning geometry in which pupils use hands-on activities with concrete objects in addition to traditional ways and using technology. In the teaching and learning of Transformation Geometry, pupils are expected to carry out tasks involving shear, stretch, enlargement, rotation, reflection, and translation on an object. In carrying out all these tasks, the pupils need to use their skills, both traditionally and technologically, in order to find out, learn, apply and communicate aspects of mathematics (JMC, 2011). In Enlargement Transformation Geometry, these tasks require pupils to identify a transformation, find or use the centre and scale factor of enlargement to perform an enlargement, as well as describing an enlargement using correct terminologies and reasonable arguments in arriving at their answers (Edexcel, 2010; OCR, 2011). Enlargement transformation maps an object onto its image through a particular centre known as centre of enlargement which could be on the point on the object, or outside the object in the plane. It could also be represented by matrix or coordinates of the image of the original shape. The image of the enlargement is proportional to the original shape (object).

Researchers studying the difficulties of geometry and transformation geometry revealed various challenges about the teaching and learning of geometry. Most studies were carried out on a relationship between the stages of cognitive development and the Van Hiele's levels of geometric conceptualization, and the results of these studies showed that most pupils were incapable of handling traditional high school geometry. The study carried out by Thomas (2003), on Pupils' understanding of Transformation Geometry concepts, showed that there was no transfer of spacial abilities for the first, second and third grades in transformation geometry problem-solving situations. This means that, in problem-solving situations, pupils at these levels of learning were unable to use spacial abilities when solving transformation tasks in the classroom. The study conducted by Gulfem and Melihan, (2017) focused on analysing the pupils' common mistakes in the 8th grade in Transformation Geometry. The survey was carried out and the results showed that pupils understood that translation transformation was a movement of replacement, but they had difficulty in the topics such as the direction of the transformation, and the position of the figure within the transformation. The study also observed that the pupils had misconceptions concerning reflection. In this case, the pupils confused similarity with congruent property of shapes. Further, Gulfem & Melihan, (2017) reported that the pupils had also problems in identifying and writing the equation of the axis of symmetry of shapes under reflection, as well as finding and using the angle and centre of rotation.

Furthermore, Ada and Kurtulus (2010) investigated the third-year university students' misconceptions and errors in transformation geometry concerning analytical geometry course given by researchers. Data were collected from seven examination questions. The result of the analysis showed that these students did not understand how to apply rotational transformation and they did not understand the geometric meaning of these concepts. In a similar study, Hollebrands (2003) investigated the nature of pupils' understanding of geometric transformations, which included translation, reflections, rotation and enlargement, in the context of the technological tool, the Geometer's Sketchpad where Pupils' conceptions of transformations as functions were analyzed. The analysis suggested that pupils' understanding of key concepts including domain, variable and parameters, and relationships and properties of transformation were critical for supporting the development of deeper understandings of transformations. According to Barbara (2011), in her work 'Treatment of Geometric Transformations' in presently available middle grades (6, 7 & 8) pupils' mathematics textbooks, that there was no consistency found in terms of order, frequency, or location of transformation topics within textbooks by the publisher or grade level. The pupils experienced serious problems when working with transformations, and this was probably as a result of the pupils receiving little support or attention, in the lessons. Further, Ilaslan (2013) reported that teachers experienced challenges when teaching transformations because they lacked self-confidence and knowledge of transformations especially in rotation and enlargement. In addition, those teachers there was no enough technological materials to use in delivering the transformation lessons. However, Jones (2011) argued that although technological materials have a vital role in education of being a tutor, tool and tutee, they are not used to full potential in mathematics. In addition, Tamara (2015) pointed out that teachers' Transformation Geometry content knowledge needed to be deepened and widened, and in particular with respect to the application of transformations, to prove that two figures are congruent or similar, because they were weak in that area.

It is further observed that nine, eleven and thirteen (9, 11&13) years old pupils' poor performance in the transformation tasks is due to non-conservation of length at an age where they are expected to have this skill (Soon, 1992). In this regard, Noraini (1998) contended that geometry instruction needed to encourage more non-routine problem-solving activities such as geometric puzzles and problems based on real-life situations to enhance geometric thinking activities. In this sense, geometric instructions also needed to be designed to encourage more interactions between teachers and learners to enhance mathematical communications.

In an earlier pursuit to find solutions to pupils' difficulties in the learning of geometry and transformations, Van Hiele (1999) argued that optimal geometry learning was achieved when pupils developed their thinking and reasoning skills. Pierre van Hiele and Dina van Hiele- Geldof investigated and proposed five levels of geometry cognition. The weak consequence of the teaching of geometry must almost entirely be attributed to the disregard of the levels (Battista and Clements, 2000). From a teaching and learning perspective, this finding seemed to be in line with the natural order of teaching and learning in which both the teacher and pupils were supposed to progress in both teaching and learning in a sequential order. Furthermore, van Hiele revealed that the learning process in geometry covered many levels, but appreciation of these levels still needed to be emphasised during teaching in the classroom. It is through the disregard of the hierarchical nature of these levels with the teacher and the pupil operating at different levels that account for much of the difficulties which pupils have in the process of learning geometry. Van Hiele asserted further that any pupil forced to a level which he or she was not ready, would only be imitating his teacher's work with no meaning. "What he heard was not integrated into his existing structures in the mind" (van Hiele, 1999). As a result, rote learning took place and little transfer occurred (van Hiele, 1999).

The above discoveries resulting from van Hiele's study have been the catalyst for much of the renewed interest in the teaching and learning of geometry and transformation geometry both in the past and present (Kekana, 2016). Van Hiele's ideas evolved primarily out of a reaction to the deficiencies perceived with the view of Piaget which says that mental development is a continuous construction comparable to the erection of a vast building that becomes more solid with each other (Piaget, 1968). The van Hiele theory is based, in part on the notion that pupils' growth in geometry takes place in terms of identifiable levels of understanding and that instruction is most successful if it is directed at the pupils' level.

Indications of pupils' difficulties in transformation geometry from studies discussed above, suggestions and ideas resulting from the van Hiele's levels of geometry understanding have provided the structures which will guide the researcher in the analysis and description of the difficulties pupils in Zambia have in transformation geometry with respect to the concept of enlargement.

METHODOLOGY

Research Design

The study adopted a mixed method design that utilised a descriptive study design. This approach was characterised by collection and analysis of quantitative data followed by collection and analysis of qualitative data (Creswell, 1998). The descriptive survey design was used because it provided a wide-ranging data that aided in-depth understanding of the pupils' thinking processes (Kombo and Tromp, 2009), which involved enlargement transformation geometry problem-solving.

Target Population

The target population comprised of grade twelve pupils in Lusaka and Southern provinces of Zambia.

Sample Size

A sample of one hundred (100) grade 12 pupils was drawn from the two schools, A and B. Fifty (50) pupils were drawn from school A, a girls' and another fifty (50) from School B, a boys' school.

Sampling Procedure

Purposive sampling was used to select the three grade 12 classes in each school. Furthermore, the study also used probability random sampling, called simple random sampling. In this technique, the class registers which were provided by grade teachers at each school, were used to select the 100 pupils, in which every second (2^{nd}) pupil in a row in each class, was selected to take part in the written test. Further, every sixth pupil in a row, from the pupils who wrote the test, were picked to take part in the interview. This technique is supported by White, (2005) and Cutis et al, (2000) who stated that the simple random sampling ensures that every element in the sampling frame has an equal chance of being included in the sample. The selection of pupils for interview was performed based on their results in the written test. These pupils were then assembled and given an overview of the interview. Kombo and Tromp, (2009) are in support of this procedure and they stated that this procedure ensured that every element that was finally chosen to take part in the study was willing and not forced to participate. Therefore, six (6) pupils, all together, from the two schools, volunteered to take part in the interview.

Data Collection Methods

The data was obtained through a written test for quantitative purposes and interview with pupils for qualitative aspects.

Data Analysis Procedure

In the analysis stage, the audio records, direct quotes from pupils and teachers and the researcher's notes were used. The responses from the pupils and teachers were transcribed and analysed for challenges experienced by pupils at each level of the Van Hiele's model, and to a question or cluster of questions. The interview questions and their analyses focused on pupils' levels of Enlargement Transformation Geometry understanding and the challenges they face in solving enlargement Transformation Geometry described. The researcher looked at pupils' written wok and their answers to the test, making meaning out of every line drawn and every pencil mark made. In addition, the researcher carefully listened to the interview records and looked at the written work simultaneously. Every pencil mark, therefore, was important for the researcher to interpret the interview accurately. Interview transcripts were carefully read, and the pupils' reasoning patterns were discovered with the help of some semantic features such as argument chains and explanations, while solving the problems. During the analysis process, the researcher also examined if certain hypothesis made about pupils' challenges were reasonable or not.

Finally, the multiple data arising from the written test and interviews from the pupils and teachers' transcripts and direct quotes and researcher's notes were analysed using thematic analysis in line with the Van Hiele's model of geometric understanding, by comparing common patterns of pupils' challenges in enlargement transformation.

PRESENTATION AND DISCUSSION OF FINDINGS

This section presents and discusses the findings of the study obtained from the written test and interviews.

Presentation of Findings

The findings are presented according to the research objectives

Difficulties encountered by pupils in solving Enlargement Transformation problems.

Figure 1 presents the results of the written test on pupils' difficulties in solving enlargement transformation geometry problems.

Figure 1: Learner's difficulties based on Van Hiele's levels

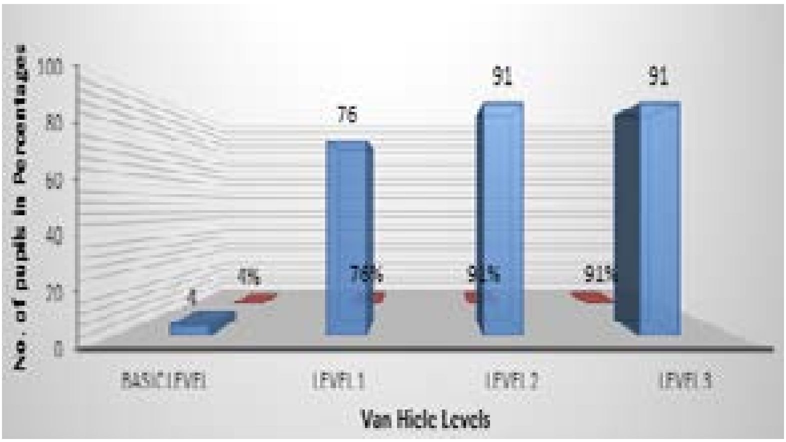

Figure 1 shows that 4 out of the 100 pupils (4%) had difficulties at the level of visualisation (linked to question 1), 76 (representing 76%) pupils experienced difficulties at descriptive level (linked to question 2) and the levels of deduction and abstraction were at par, with 91 pupils (representing 91%) having experienced difficulties at these levels (linked to questions 3 and 4 respectively). It is noticed that most of the grade 12 pupils at the two schools were reasoning at the lowest level (visualisation) of the Van Hiele's model. The results show that the grade 12 pupils had problems and difficulties in solving enlargement transformation geometry especially with those problems of higher order levels (description, analysis and deduction) of the Van Hiele's model of geometry thought.

Root causes of the challenges that pupils face in solving enlargement transformation geometry problems.

From the pupils' end, the researcher embarked on trying to establish, from the pupils themselves, the challenges that they faced in solving enlargement transformation geometry problems. The researcher interviewed 6 of the pupils who experienced more difficulties during the written test so that he could have a clear and in-depth understanding of the results at hand. The questions asked during the interview session were focused on the areas they experienced problems with during the written test. Each pupil was given a piece of paper, a pencil and a pen. They were required to explain, verbally or in writing, how they arrived at their answers and they could also give reasons for their answers. All the six pupils interviewed were able to recognise the figure as an enlargement but had difficulties in describing the enlargement fully. They also expressed challenges with a question which required them to transform triangle ABC about the centre with scale factor -2 and explaining the steps they took to obtain the image. For example, student B15 said that the transformation was enlargement and kept quiet. However, the researcher requested him to use the properties of the centre and scale factor of enlargement to fully describe the transformation, but he remained quiet and shook his head indicating that he did not know the process fully.

Findings from the interview on the use of the terms centre and scale factor to describe the transformation mapping triangle R onto triangle X revealed the following findings.

Pupil B15: *"Maaa-aa! I don't know, I only know that triangle R has been made bigger into triangle X."*

Pupil G21: *"I know that there should be the centre and scale factor to enlarge a figure, but I don't know how to find them or describing enlargement now…." "Besides that, our teacher told us that transformation is not important and we should not answer a question on transformation in our final examination."*

Other respondents failed to even understand the question they wrote in the test. This can be confirmed from one interviewee's response when he was asked to enlarge triangle ABC about the given centre (1.5, 1) with scale factor -2. He stated

Pupil B 30: *"What do you mean sir?"*

The interviewer clarified that he meant the steps the student would follow to transform triangle ABC under enlargement about the centre (1.5, 1) and with the scale factor (-2). The response was that:

Pupil B30: *"Oh… you move each point two steps away from the centre to find all the points on the image. But here sir, do not ask me another question please because these things confuse me a lot."*

Another pupil had this to say on the same question:

Pupil B31: *"I have to move it to here (pointing at the position calculated, but an incorrect one).*

The interviewer probed further by asking if it was the same procedure he used to obtain the image of point A."

Pupil G31: *"Yes"*

The researcher asked the pupil to transform points B and C using the same procedure she followed to transform A.

Pupil G31: *"Okay sir…. But it looks like something else again."*

The demonstration she showed resulted into an incorrect image of triangle ABC.

The researcher asked why there was a need for the centre and scale factor in enlarging a figure.

Pupil G31: *"I think I am not sure here, help me, please."*

The respondent could only manage to give a relevant response after she was guided on what to do.

From the findings above, it can be deduced that most pupils faced challenges in solving enlargement transformation geometry problems regarding determining the centre and scale factor of enlargement. It was also noticed that most pupils had challenges concerning constructing an image under enlargement using the centre and scale factor of enlargement, and they were challenged with using the concept of enlargement to carry out geometrical proof, in which they failed to identify the relationship between the figure and its image under enlargement transformation. Furthermore, the findings showed that the pupils lacked concept of size and shape and proportional sides, and they also lacked understanding of geometry terms used.

DISCUSSION OF FINDINGS

The first objective of this study was to ascertain pupils' levels of difficulty in enlargement transformation geometry learning. The study demonstrated that most of the grade 12 pupils were reasoning at the lowest level (visualisation) of the Van Hiele's model of geometric thought. The study findings indicated that 96 (96%) of the pupils provided correct solutions to question

1 linked to the basic level. These pupils identified the pair of figures under the action of enlargement from the set of pairs of other transformations. According to Van Hiele (1999), pupils are classified as having achieved the basic level of geometric thought if they are able to visualise and name transformation by actual motion and by using standard or no standard name. In this regard, it implies that most of the grade 12 pupils had pre-requisite knowledge of basic properties of different shapes undergoing a particular transformation. This made it easier for them to identify the orientations of images of given transformations as well as being able to classify actions and motions performed on figures to give rise to the appropriate images (Clement and Battista, 1992). Furthermore, Kate et al (1999) argued that if pupils are able to identify and perform enlargement informally, then they are ready to move to Van Hiele's analysis level, where visual skills are the basis of progression to the higher levels. It can also be argued that the grade 12 pupils at this level had mental abilities to form and manipulate objects visualised under enlargement, implying that they had earlier experienced rich manipulation of physical objects (Ibid, 1999).

The results of pupils' performance under level 1 leaves much to be desired. Question 2 in the written test examined the extent to which the pupils could use the properties of change of enlargement to transform figures and to relate the image to the figure under an enlargement. The study showed that 76% of the pupils had problems with the concept of the centre and scale factor of an enlargement. These pupils experienced difficulties to provide the correct centre and scale factor. The study further demonstrated that if a pupil was unable to find the centre of enlargement, then that pupil could also fail to find the correct scale factor of enlargement. This could be attributed to pupils' lack of understanding of the concepts of the 'centre' and 'scale factor' of enlargement. Furthermore, the study findings showed that most of the grade 12 pupils could not demonstrate that the scale factor of enlargement is the ratio of lengths of corresponding sides of the image to the object figure, or the ratio of the corresponding distances of points on the image from the centre of enlargement to the distances of points on the object figure from the centre of enlargement. The findings are consistent with the ECZ (2012) and OCR (2012) reports that pupils performed poorly in transformation geometry. In addition, the study showed that the pupils had problems in identifying properties that the corresponding sides of the image and the object were proportional and also identifying properties that correctly described Enlargement Transformation geometry. In this regard, it can be asserted that these pupils lacked pre-requisite knowledge of similarity of figures, where they were expected to be knowledgeable of concepts of the size, proportional lengths and shape. In this regard, OCR (2012) puts it clear that most of the pupils fail to use these properties of enlargement because they cannot even appreciate the roles of the centre and the scale factor of enlargement. It can be argued that the pupils could have achieved this level if they had the knowledge about the concept of ratio and proportion. The study was also informed that most pupils lacked the analytical skills that required them to use simple words relating to transformation to analyse a given figure and its image after enlargement. These findings are in agreement with Kate et al (1999) who explained that if the pupils have not achieved level 1 then such pupils lack spacials skills, such that they may have problems in visual imagery and mental diagrams, as they attempt to form and manipulate figures visualised. Such pupils would not have formidable necessary geometric skills for the higher level transformation.

The study also demonstrated that most of the pupils in grade 12 had difficulties in using the properties of enlargement to perform enlargement transformation. The findings showed that 91% of the pupils could not achieve level 2 since they could not correctly answer question 3 in the written test. The pupils experienced problems and difficulties to enlarge the figure using the given centre and scale factor of enlargement as well as being unable to give proper and full description of an enlargement. Further findings revealed that pupils were unable to create meaningful definitions and formal arguments to justify their reasons for the action of enlargement transformation. The study findings are in line with Van Hiele (1999) who postulated that pupils who fail to perceive relationships between properties and between figures are considered to have not achieved higher order level problems of level 2. However, Hoyles and Jones (1998) & OCR (2012) explained that the pupils who cannot properly visualise external constructions of figures described by cartesian coordinates usually fail to mentally manipulate points of the cartesian plane to form figures under enlargement. In this regard, it is possible to argue that the pupils who experienced difficulties and problems achieving level 2 question mighty have lacked the basic knowledge of the concepts of ratio and proportion and they could have perhaps missed the concept of the properties of enlargement. It is not surprising that these misconceptions regarding enlargement, pupils struggle with the concept of similarity, where many of them give unreasonable solutions to the problems (Edexcel, 2012).

Another notable result from the study was pupils' weakness and failure to identify the profound relationships of figures under enlargement, in their quest to do proof of the concept of enlargement. The results showed that 91% of the pupils did not provide correct solutions to question 4, and thereby failing to achieve level 3. The pupils failed to demonstrate their knowledge of the concept of size and shape and proportional lengths regarding similar figures. The results are consistent with Jones (2002) who argued that many pupils experience difficulties with writing proofs and most of them are unsuccessful in solving geometry problems. The study also showed that the pupils experienced geometrical language problems as they were trying to analyse geometrical situations and relationships between and among figures. In addition, Pickereign, (1996) contended that language associated with geometry and transformation geometry is crucial for children to acquire a more complete understanding of geometry concepts.

The other objective which the study addressed was to determine the root causes of the challenges that pupils face in solving enlargement transformation geometry problems. Data was collected from the interview that involved 6 pupils. Firstly, the study was informed that those pupils had challenges with visually identifying the effects of enlargement transformation on figures. The pupils demonstrated ignorance of the basic relationships of figures under enlargement through identifying the incorrect image of triangular figure given and they decided on the image of a different (stretch) transformation. The study findings agree with Clements and Battista (1992) who contended that pupils who fail to solve geometry problems at visualisation level have limited ability to even notice a subset of the visual characteristics of a shape, resulting in an inability to distinguish the figures. In this study, the pupils who could not meet the criteria of achievement under the basic level of the Van Hiele's model had no enough pre-requisite knowledge of basic properties of enlargement transformation.

The study also demonstrated the pupils interviewed had challenges in verbally describing the enlargement transformation fully. Furthermore, the pupils expressed challenges in explaining the steps they took to obtain the image after enlargement. However, these pupils were able to mention that the transformation was enlargement and kept quiet since they could not explain how to use the properties of the centre and scale factor of enlargement to fully describe the transformation. The study findings are supported by Lappan (1999) who noted that such deficiencies resulted into pupils' inability to apply geometric terminologies in describing a figure and its image and thereby failing to solve enlargement transformation problems. It can be further argued that geometry language especially in the comprehension of geometric terminologies play a key role in learning and understanding of geometric concepts (Clements & Battista, 1992).

The study findings established that the respondents had challenges regarding proving the application of the concept of enlargement. They were also challenged to use some concept like the size of angles being equal, the preservation of shape, and the difference in size of shape and proportional lengths in their arguments to show that both triangles in question were similar. This could be so due to the pupils' inabilities to comprehend the concepts imbedded in enlargement and the transformation terminologies (Clements, 2001). As much as they could cite that the triangles under enlargement transformation had the same shapes, some of these pupils (for example the pupil coded B2) could not tell why the triangles under enlargement had the same shape but different in size. For instance, Pupil B46 mentioned that he did not know how to put it when he was asked by the interviewer, to explain why the two triangles under enlargement the same shape had but different sizes, while Pupil G53 said that she had no authentic idea. This could be as a result of these pupils' failure to interrelate the properties of change to the figure due to enlargement (Van Hiele, 1999). According to Cox (2012), these pupils could have not been supported in some ways to solve enlargement by not, first of all, exposing them to solving similarity problems as well as solving other problems involving ration and proportion.

Further, the study also showed that pupils had little or no understanding of geometry terms used. In this regard some pupils did not know the meaning of the terms "mapped unto", "similar" and "proportional". Some pupils also used their own terms such as “made big” and “move”. Most of the pupils could express ignorance of some suitable geometric terms rendering them to misunderstand the task at hand, and the researcher had to rephrase statements to make them understand what was being talked about. It was evident that geometry language, especially in the comprehension of geometry terms, plays a key role in learning and understanding of geometric concepts (Baston et al, 2010). In this sense it is therefore important that pupils should understand the geometry and transformation geometry language and terminologies before they could attempt to solve the problems, meaning that language associated with geometry and transformation geometry, is crucial for children to acquire a more complete understanding of geometry concepts (Pickereign et al, 2000).

CONCLUSION AND RECOMMENDATIONS

Conclusion

From the discussion in the previous captioned heading, it is noted that most of the Grade 12 pupils were reasoning at the lowest level of the Van Hiele’s model which is visualization and had difficulties at the level of description (level 1), abstraction (level 2) and deduction (level 3). Furthermore, the study showed that most pupils experienced difficulties as the levels of transformation geometry problems increased in scale. Further, most pupils were challenged with solving enlargement transformation geometry problems regarding determining the centre and scale factor of enlargement as well as constructing an image under enlargement using the centre and scale factor of enlargement. The study also concluded that most of the pupils had challenges regarding the use of the concept of enlargement to carry out geometrical proof in which they failed to identify the relationship between the figure and its image under enlargement transformation. In this vein, the pupils demonstrated lack of knowledge regarding the concept of size and shape and proportional sides. It is also noticed from the study that pupils had challenges with the understanding of geometry terms used and this deficiency resulted into pupils' inability to apply geometrical terminologies when describing figures under enlargement. Finally, the study concluded that most pupils lacked relevant and enough pre-requisite knowledge to enlargement transformation, and they lacked exposure to more geometrical problem-solving situations.

Recommendations

Arising from the conclusions presented in this study, the following recommendations were made to solve the challenges at hand:

(i) Teachers should strive to motivate learners on the topic and equip them with necessary pre-requisite knowledge on enlargement transformation. This should include introducing learners to practical situational problem-solving tasks which should involve explanation as well.

(ii) The secondary school mathematics curriculum should be appropriate for the various thought levels and guiding pupils to learn about significant and practical concepts that would require pupils to explain and justify their ideas and refine their thinking.

(iii) Teachers should be vested with modern teaching approaches and tools so that transformation geometry concepts could be easily comprehended by pupils through visual actions, illustrations and interpretations. This will help in instilling and developing pupils' geometrical concepts and language, among other key valuables necessary for geometry learning.

Recommendations for future research

The topic, ''*Relevance of geometry and geometrical language in the learning of transformation geometry''*

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